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KEYNES' AND THE SANTA FE INSTITUTE'S COMPLEXITY: SAME CONCEPTS, DIFFERENT METHODS?

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ABSTRACT

Keynes' 'fallacy of composition' and, later, the 'Santa Fe perspective' have shown that the economy may be defined as a "complex adaptive system", since it is not something given but results from constantly developing interactions among heterogeneous individuals. Endorsing the definition of 'dynamic complexity', the paper analyses the extent to which Keynes' notion of complexity is coterminous with the one developed at the Santa Fe Institute. An analysis of the role of expectations in the *General Theory*, along with the dimension of time, history and path-dependence, makes it possible to conceive Keynes' work as an open, complex evolving system and to establish a direct connection with the 'Santa Fe perspective'. However, Keynes' "two-stage methodology" which allows the analyst to deal with complex organic material without resorting to reductionism by means of ordinary language involves the inherent non-measurability of complex magnitudes which does not depend on the individual's limited ability to measure the complex object. This feature may cast some doubts on the inheritance of Keynes's methodology by the 'Santa Fe perspective', which attempts to handle complexity through simulation techniques implicitly based on a sophisticated mathematical formalism.

Keywords: Keynes, Methodology, Complexity, Santa Fe perspective, Complex systems.
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Introduction

Keynes's 'fallacy of composition' and, later, the 'Santa Fe perspective' on spontaneous emergence of higher-order structures from lower-order

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ones have shown that the economy is not something given but results from constantly developing interactions among heterogeneous individuals which can be properly described as a complex system. Since Seth Lloyd, a physicist at the MIT, has gathered forty-five different definitions of complexity (Horgan 1997: 303), it is of primary importance to state clearly which of them is being endorsed here. Curiously, the three most frequently used notions of complexity in economics – 'general complexity', 'dynamic complexity' and 'computational complexity' [1] – do not appear in Lloyd's list. Given that, on the one hand, many economists when referring to complexity have the notion of 'dynamic complexity' in mind, and on the other, that it resembles the concept of a "complex adaptive system" adopted by the 'Santa Fe perspective'(Arthur *et al.* 1997a); in what follows this definition is adopted.[2]

In the introduction to *The Economy as an Evolving Complex System II* (Arthur *et al*. 1997a), the authors, while criticising the standard conception underlying neoclassical economics, define a "complex adaptive system" in terms of John Holland's (1988) "adaptive nonlinear networks", whose distinguishing feature is their ability to "anticipate" rather than "act simply in terms of stimulus and response". This conception of economics, as well as the economy, has several advantages in terms of both "cognitive foundations" and "structural foundations". Firstly, economic agents are not assumed "to have common knowledge about each other and rational expectations about the world they inhabit". Therefore they do not act as perfectly rational optimising agents; rather, they "make sense" of their problems in order to solve them through the adoption of a "variety of distributed cognitive processes" by means of learning and adaptive algorithms (genetic algorithms, neural networks and classifier systems, to name but a few). Secondly, agents are presumed to inhabit a world made of many morphologically diverse and interconnected parts in which "units at one level combine to produce units at the next higher level" and whose structure is not presumed to be hierarchically organised. Therefore, the removal of one part induces the overall system to self-adapt due to the changed circumstances (Arthur *et al*. 1997a: 4-6). Moreover, the complexity of the system is considerably increased by the heterogeneity of individuals in terms of skills, preferences and experience, on the one hand, and the absence of an 'invisible hand' made possible by nonlinear dynamics, on the other. Because the actors of the system are clustered around a variety of groups and institutional structures which operate on different temporal and spatial scales,

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1 An account of these three definitions is provided by Holt *et al.* (2011: 361-364).

2 Richard Day (1994) argues that a system is dynamically complex if, for endogenous reasons, it fails to converge to a point, a limit cycle or a smooth explosion or implosion. Such systems can generate endogenous discontinuities in system variables.

they generate out-of-equilibrium dynamics which make it impossible to derive the aggregate behaviour from the sum of the behaviors of individual components even in the presence of an array of activities carried out by the participating sub-components. On taking these features into account, it is possible to comprehend why of crucial importance are the endogenously generated mechanisms according to which a specific configuration of the system emerges, rather than the final outcome considered alone.

With this definition of complexity in mind, the paper analyses the extent to which Keynes' notion of complexity is coterminous with the one developed at the Santa Fe Institute. The analysis first concentrates on the notion of complexity as envisaged by Keynes, which rests upon his organic conception, as well as his view of the economy, and on a non-mathematical methodology aimed explicitly not to contradict the essence of the complex economic material (Section 1). The focus then shifts to the analogies that arise on conceiving the *General Theory* as an open, complex evolving system. In this respect, it is argued that the 'Santa Fe perspective' has inherited Keynes's core concepts of expectations and individual behaviour with respect to their process of formation and evolution as well as the open and path-dependent nature of Keynes' masterpiece (Section 2). By contrast, Section 3 discusses the discrepancies between the two views of complexity. It takes into account the different methodologies provided by Keynes and the Santa Fe Institute to deal with complex material; methodologies which seem ultimately to diverge in terms of the nature of models and the crucial role assigned to mathematics. Finally, the paper draws some conclusions and suggests a possible way forward (Section 4).

1. Keynes's Notion of Complexity

The notion of complexity envisaged by Keynes reflects his conception of the economy as made of intrinsic interdependencies among individuals and variables which cannot be broken up into different parts without losing the coherence of the system as a whole.[3] In particular, Keynes's defini-

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[3] Drawing on the methodological insights provided by Carabelli (1988), we adopt an organicist approach to the definition of complexity which relies on the organic interdependence among variables. However, there has been a lively debate among scholars as to whether Keynes' position in *A Treatise on Probability* [Keynes 1973b (1921)] is atomistic or organicist (see Bateman 1987; Carabelli 1988; Davis 1989; O'Donnell 1989 and Marchionatti 2010). Independently from the atomic or organic stance adopted, it cannot be neglected that Keynes is "uninterested in forms of *interdependence* between economic agents" that, "where pursued, is neither reminiscent of the organicism Keynes allows of individual minds, nor incompatible with Keynes's atomism of the individual" (Davis 1989: 1169, emphasis added).

tion of complexity originates from his close concern for the 'philosophy of magnitude'. Indeed, the concepts "complex or manifold" are defined by Keynes as "capable of variations of degree in more than one mutually incommensurable direction at the same time" (Keynes 1971 [1930]: 88).[4] Because of the multidimensional and heterogeneous nature of complex magnitudes, which prevents comparisons on a numerical basis, the issue of complexity turns out to be one of measurement, or to be more precise the absence of a common unit of measure according to which all relations relative to the quantity under consideration may be expressed.[5] Thus, limitations of measurement do not depend on the epistemic skills of the cognitive subject but, on the contrary, on the peculiar features of the object to be measured. For this reason, Keynes firmly warns the scientist that complex magnitudes, being non-numerical quantitative concepts, "cannot in themselves provide the material for a quantitative analysis" [Keynes 1973a (1936): 39]. In Chapter 4 of the *General Theory*, devoted to "The Choice of Units", Keynes explicitly writes:

> The three perplexities which most impeded my progress in writing this book, so that I could not express myself conveniently until I had found some solution for them, are: firstly, the choice of the units of quantity appropriate to the problems of the economic system as a whole … (*ibid.*: 37).

Given the aforementioned difficulty of measurement, we should ask ourselves how complex material can be treated. According to Keynes, even though complex magnitudes are theoretically vague concepts, they are not logically indefinable. "Conundrums … of no solution", are in truth not insurmountable once economists renounce using "quantitatively vague expressions" (*ibid.*: 39). In fact, despite the impossibility of finding a unit of measure, we are enabled to deal with complex magnitudes by rejecting the 'atomic hypothesis' and by resorting to an alternative tool, a "two-stage methodology" (Carabelli and Cedrini 2014). Notably, in the first stage we have to examine the various economic variables "by isolating the complicating factors one by one". Then, in the second stage, "after we have reached a provisional conclusion we then have to go back on ourselves and allow, as well as we can, for the probable interactions of the factors amongst themselves" [Keynes 1973a (1936): 297]. However, we might reasonably ask how to move from the first to the second stage of the analysis. The answer to

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[4] For a detailed description of complex magnitudes see Carabelli (1992; 1994; 1995).

[5] It is worth quoting, in this respect, what Carabelli (1992: 27, fn. 6) claims: "If attention were paid to Keynes's critique of measurement […] the Cambridge controversy on the measure of capital would have been unnecessary".

this question resides in Keynes' attribution of the task of overall control to ordinary language rather than formalised language, for its open logic generates knowledge which is imperfect, partial or vague because it concerns an organic whole of which complete, precise knowledge is impossible. In contrast to the strictures of classical logic, which attempts to generate precise knowledge, ordinary logic recognises the interconnectedness of organicism but does not regard it as impenetrable or inextricable. It therefore provides the basis for procedures to segment interconnectedness in order to generate knowledge (Carabelli 1988; Gotti 1994; Chick and Dow 2001). Therefore, by interpreting the connection between variables as only 'probable', and by allowing the account to be kept open for further qualifications, ordinary language overcomes those imperfections that mathematical generalisation reveals in the economist's way of thinking, allowing the latter to not "lose sight of the complexities and interdependencies of the real world in a maze of pretentious and unhelpful symbols" [Keynes 1973a (1936): 297-298]. Now that the notion of complexity as envisaged by Keynes has been analysed, let us turn to assessment of the analogies between Keynes and the 'Santa Fe perspective'.

2. Keynes and the Santa Fe Institute: the *General Theory* as a Complex System

"Picturing the macro economy as a complex system and a depression as the result of the intricacies of complex dynamics may have been Keynes' visionary idea behind the *General Theory*". Thus Colander (2011: 190) depicts Keynes' masterpiece. Also Chick (2004), Marchionatti (2010) and Carabelli and Cedrini (2014) emphasise the complex nature of Keynes' economics, which is identified in the "open-ended" structure of his major work with respect to both its expository form (Gotti 1994), made of recurrent references back and forth to other parts of the book, and the economic system which it represents: the relation between the micro and the macro structure through the role of time, history and path-dependence should not be taken as given. To conceive the *General Theory* as an open, complex evolving system, we must analyse the role of both short-run and long-run expectations.[6]

[6] An open system is a network made of several interrelated parts which interact with other systems or the outside environment. A complex system is open because the interactions make determining its border difficult: agents act in order to reproduce and reinforce the system, leading, eventually, to its continuous evolution and adaptation to changing circumstances. For a taxonomy of the main features of an open system see Dow (2002: 140).

Endogenous Expectations and Irrational Behaviours

"Short-term expectations" relate to the decision about how much to produce in the immediate future with a given level of capital stock. They are formed by firms not through complicated optimising decisions but rather on the assumption that "the most recently realised results will continue, except in so far as there are definite reasons for expecting a change" [Keynes 1973a (1936): 51]. Differently, "long-term expectations" are related to decisions on whether or not to change the amount of capital stock, that is, investment, in an overall context characterised by limited knowledge of the environment available to the actors. Because the agent's degree of optimism or pessimism about the future is based on a sentiment, the long-run expectations of firms appear to consist of two components: "waves of irrational psychology", which alter the sentiment across the network of firms as a whole, and the "outcome of mass psychology of a large number of ignorant individuals" driven by a "spontaneous urge to action rather than inaction" – the 'animal spirits' [Keynes 1973a (1936): 154, 161]. Given their irrational behaviour and thus the impossibility of computing their conduct as the "outcome of a weighted average of quantitative benefits multiplied by quantitative probabilities" (*ibid.*: 161), they can only react to changing circumstances by imitating others' behaviour. Therefore, as Ormerod (2009: 28) suggests, agents act on a network which at any point in time is in "one of k states of the world, where k is the degree of optimism/pessimism" and follow a threshold rule by which they alter their state of the world according to that of their neighbours. This aspect is crucial to the notion of complex system because it shows how individual expectations are not formed in isolation but in relation to an interconnected set of agents by means of "adaptive non-linear networks". Indeed, the unintended consequences of individual behaviours produce an aggregate outcome at the level of the whole system which feeds back on and affects the nature of interactions. In turn, these changed interactions at the level of the system's parts produce a new aggregate outcome which generates positive feedbacks in a continuing dynamic between action on these two levels. However, there is another aspect which generates further instability in the system so that the micro-structure transforms, and is subsequently transformed by, the macro-structure. Expectations, according to Keynes [1973a (1936): 315], are based on "shifting and unreliable evidence". This highlights the importance of fundamental uncertainty, not reducible to probabilistic reasoning, as an additional source of complex dynamics (see Rosser 2006; Davis 2017)[7] and

[7] We do not deal here with the thorny issue of the ontological or epistemological founda-

it directly relates Keynes' analysis to two paramount features of the 'Santa Fe perspective': the primacy of agents' behaviour driven by emotional considerations concerning constrained optimization rules, on the one hand, and the impossibility of generalising from micro decisions to macro outcomes by means of a reductionist approach – Keynes' 'fallacy of composition' – on the other.

The above-described mechanisms are well represented by Keynes' 'beauty contest' in which reflexive and interdependent processes dominate the way in which each individual takes decisions: when different people have different views about each other's expectations, the results can be dynamically complex, even when some of the parties may actually possess rational expectations (see Davis 2017). They are also confirmed in the work by Brian Arthur and other colleagues at the Santa Fe Institute (Arthur *et al.* 1997b) who study how the effects of decision-making by a group of agents depend on what other agents do. Rejecting common knowledge as a starting point of their analysis, they build a theory of asset pricing in which agents generate expectational models about price movements in the market by means of "interpretative devices". Once these models are generated, they are tested by trading and discarded if not successful. Since agents derive their expectations from a completely inductive decision-making process – an imagined future that is the aggregate result of other agents' expectations – the authors conclude that interactions lead to the emergence of endogenous, self-referential expectations generated by continual evolution and adaptation of the system to the market created by expectations themselves. In other words, an "evolving ecology" of interacting "interpretive devices" arises by means of agents that use them to generate their expectations.[8] Given the deductive indeterminacy of the latter, which is proved by the authors in their model, agents have to cognitively interpret the world that they inhabit by building "models of the economy and act on the basis of predictions generated by these models" (Arthur *et al.* 1997a: 4). The complete absence of any optimizing rules for agents to follow, which does not depend upon their epistemic skills but rather on the impossibility of defining which optimal course of action to undertake, is also evident in a well-known study by Arthur (1994) on the "El Farol bar problem". The way agents form their expectations is once again self-referential and relies on in-

tions of fundamental uncertainty, but refer the reader to the recent debate on ergodicity versus non-ergodicity among O'Donnell 2014; Rosser 2015 and Davidson 2016.

[8] It is interesting to note how the authors discover that speculative bubbles, technical trading and persistence of volatility spontaneously emerge from agents' interactions, as well as that homogenous rational expectations become a special case, possible in theory but unlikely to occur in practice.

ductive reasoning: as each choice alters the performance of other agents, an adapting and evolving ecology of expectations emerges from interactions.

Time, History and Path-dependence

Expectations are related to another important feature of an open system: the temporal dimension. Keynes himself admits that "it is in the nature of expectation that it takes account of the time element" (Keynes 1973c: 52). However, to understand how the notion of time relates to complex systems, consider the following argument.

Carabelli and Cedrini (2016) argue that Keynes' conception of time is inseparable from change. Indeed, if on the one hand the passage of time constantly changes how the past and the present relate to one another, on the other it connects them because "agents' actions in the past provide the grounds from which they adjust their subsequent behavior" (Davis 2017: 18). Therefore, if we recall the above-described definition of complex dynamics, we may acknowledge that a view of time not conceived in terms of a given spatial magnitude which merely brings the future back to the present (as state-contingent choices in general equilibrium theory) may properly account for interactions and self-reflexive relationships central to the idea of an economic system in continuous evolution and adaptation to processes of change. Yet this analysis is complicated by whether or not short-run expectations should be considered fulfilled. If they are, and thus agents do not constantly adjust their behavior, is there any room for complexity? To answer this question we should bear in mind the analysis carried out by Chick (2004).

Time manifests itself through the influence of history, which may be allowed to enter the picture through "the technique of partial and provisional closures" that give rise to "a tension between timefulness and timelessness" (*ibid.*: 12). Given the lack of simulation tools in his time, Keynes was forced to segment the complex material under investigation by means of a short-period equilibrium analysis which leads to the creation of several subsystems in which short-run expectations are considered to be fulfilled and time suspended. However, this segmentation does not prevent the openness of the system because it does not foreclose path-dependence: given the limitless possibilities, a precise path is not chosen. Even when it is not possible to say where the system will end up, provisional closures make it possible to extract a specific configuration of the system about which a good deal can be said. In other words, if all the interactions among variables can be known, then the *General Theory* becomes a path-dependent dynamic system. On the contrary, if these interactions remain unknown, resorting to the temporary closure permits an equilibrium to be found amongst all the possible outcomes, and the interrelations of which the theory is composed can emerge (Chick and Dow 2001: 713). In this regard, it should be noted that, because of the sensitivity of small changes to initial conditions which prevents any kind of prediction, thorough understanding of their effects on the overall structure of complex systems is far from achievable. However, lack of predictability does not completely hamper explanation. Therefore, the practical expedient of segmentation originally introduced does not prevent Keynes from capturing the path-dependence of the economy and portraying the latter "not as deterministic, predictable, and mechanistic but as process dependent, organic, and always evolving" (Arthur 2014: 187).

The foregoing discussion suggests that the theory of Complexity Economics developed at the Santa Fe Institute has inherited Keynes's complexity vision with respect to the process of formation and evolution of "motives, expectations, psychological uncertainties" (Keynes 1973d: 300) underlying Keynes' conception of the actors of the economy; a conception which, in the words of Arthur *et al*. (1997a: 3, emphasis in the original), on the one hand attempts "to accommodate the distinction between *agent*- and *aggregate*-levels" and, on the other, "accounts for the emergence of new kinds of relevant state variables [...] new entities, new patterns, new structures". Because of the focus on the process – and not just the outcome – through which a particular structure emerges, the 'Santa Fe perspective' is also known as the "process-and-emergence perspective" (*ibid*.). There follows detailed analysis of the substantial inconsistencies between Keynes and the 'Santa Fe perspective'.

3. Keynes Versus the Santa Fe Institute: How to Deal with Complex Material

Discussion of the discrepancies between Keynes and the Santa Fe Institute requires correct understanding of the methodology that they developed. This will make it possible to specify their implications in terms of the nature of models and mathematical formalism. Let us start with the nature of models.

The Nature of Models

Generally speaking, a model can be based either on verbal argumentation or on mathematical equations. Verbal argumentation is highly flexible and adaptable to the phenomenon under study, even though it prevents formal calculations and tests from being performed. By contrast, equation-based models allow for computations and testing, but, because of the nonneutrality of mathematical formalism, they require crucial assumptions about the nature of the object of study if connections between theory and the real world are to be made (Chick and Dow 2001). Indeed, mathematical tractability limits the complexity of scientific models. When differential calculus was the only approach available for modeling, models had to be kept simple enough to be mathematically solvable. However, with the advent of computer simulation, the limitation of mathematical tractability was removed and problems requiring less simplified models started to be addressed by including more complex features of the real world.

The methodology developed at the Santa Fe Institute – 'agent-based modelling' – is thoroughly based on computer simulations. "As in a culture-dish laboratory experiment", Tesfatsion (2006: 837) writes, the modeler "starts by computationally constructing an economic world comprising multiple interacting agents" represented by "an encapsulated piece of software that includes data together with behavioral methods that act on these data". In effect, in order to be able to capture the complexities of the real world – such as how individuals and the environmental variables that affect them vary over space, time, and other dimensions – models have to include processes known to be important but are too complicated to be solved analytically. For this reason, the modeler steps back to observe the development of the world over time once the simulation has been run and the results shown. Differently, Keynes's method is centred on the idea that any model needs to have a *logical* nature and, at best, would yield only a partial, highly imperfect picture of the economic reality. In Keynes' own words (1973A [1936]: 297), "the object of our analysis is not to provide a machine [...] which will furnish an infallible answer" but rather a guide to users' judgment in the form of "an organised and orderly method of thinking out particular problems". Because Keynes' non-positivistic methodology is contingent upon contexts of shifting cognitive circumstances normally occurring under conditions of limited and partial knowledge, it is ultimately intended to help economists to avoid logical fallacies in reasoning; that is, not to resort to the reductionism characterising orthodox economics (see Carabelli 1988; 1991). This conception of economics has crucial implications for social scientists. Firstly, since economics is defined as "a science of thinking in terms of models joined to the art of choosing models which are relevant to the contemporary world" (Keynes 1973d: 296), we ought to ask ourselves how a relevant model should be constructed. Given that it does not emerge automatically from empirical study as a result of the blind manipulation of data, but rather from its ability to concentrate attention on the relevant factors, we have to accept limitations in our ability to develop a complete scientific model of the economy while making a decision on what part of concrete reality to incorporate in the model itself

(Marchionatti 2010: 134). Keynes terms this decision "judgement of value", and it is what makes economics into an art because of the need to exercise the art of introspection in order to study mental processes.[9] Therefore, according to Keynes, the choice of models is more coterminous with an art than a pure science. To an economist, this means that an important skill to carry out his/her task is required: what Keynes terms "practical intuition" and defines as "an unusual combination of keeping an open mind to the shifting picture of experience and of constantly applying to its interpretation the principle of formal thought" [Keynes 1972 (1933): 108]. In turn, as models do not provide "a body of settled conclusions immediately applicable to policy" (Keynes 1983: 856), before a follower of Keynes' method moves directly from economic theory to policy s/he must always keep at 'the back of his/her mind' a meta-model of the relevance of other models and adjust the specific model's conclusions to fit the *quaesitum* raised by the circumstances (see Colander 2011). Is this conception of models the same as the one developed by the 'Santa Fe Perspective'? To answer this question, we have to concentrate on the role of mathematical formalism.

Mathematical Formalism

As previously noted, every agent-based model is a computer program. Thus, it could be computed by a Turing machine for which there exists a unique corresponding and equivalent partial recursive function and cast as an explicit set of extremely difficulty mathematical formulas – recursive functions (Epstein 2006: 1590-1591). In light of the opposition between analytically solvable mathematical models and simulation models, is the distinction between agent-based and equation-based models a mere illusion? Let us consider the following argument put forward by Epstein (*ibid.*: 1591, emphasis added):

> To all but the most adept practitioners, the recursive function representation would be quite unrecognizable as a model of social interaction, while the equivalent agent model is immediately intelligible as such. However, at the dawn of the calculus, the same would doubtless have been true of differential equations. It is worth noting that recursive function theory is still very young, having developed only in the 1930s. And, it is virtually unknown in the social sciences. It is the *mathematical formalism* directly isomorphic [...] to computer programs, and over time, we may come to feel as comfortable with it as we now do with differential equations.

This declaration casts several doubts on the indictment that agent-based models are just simulations for which no equations exist (to prove

9 See Keynes' letter to Harrow, 4 July 1938 (Keynes 1973d).

his arguments Epstein (*ibid.*) refers to several agent-based models that have been mathematised with familiar techniques). In effect, the indispensable role that mathematics carries out for the 'Santa Fe perspective' is also made clear by Arthur *et al.* (1997a: 12-13) when they argue that "while a few of the papers completely avoid mathematics, most of the papers do present mathematical models – whether based on statistical mechanics [...] or agent-based computations. Yet sometimes the mathematical models the authors use leave important questions unanswered". This should not be surprising if we consider that, according to the analysis carried out by Fontana and Corsatea (2013) on the working papers issued by the Santa Fe Institute over a time span of fifteen years (1989-2004), an important driver of new ideas is the interdisciplinarity of research, and the main competences of scientists working at the Institute pertain to the fields of economics, physics, computer science and mathematics. Hence, whilst on the one hand the endeavour of Arthur and his colleagues to resolve "the inherent tension between traditional mathematical tools and phenomena that may exhibit perpetual novelty" (Arthur *et al.* 1997a: 13) seems to perfectly conform with the argument put forward by Epstein, on the other it raises issues regarding the knowledge of mathematics and the computational language necessary to deal with agent-based computational economics. Does this clash with Keynes' conception of economics and complexity?

As a mathematician by training, Keynes appreciated the mathematical contributions to economics as long as they shed light on economic problems. This concerns those exceptional circumstances in which the material under study is not 'complex' and ordinary language logic can make room for formal logic (Carabelli 1988: 149). However, when the complexity of the subject increases, the use of mathematical techniques considerably restricts the nature of economic material characterised by an increasing number of variables. Accordingly, the issue at stake turns out to be that of broadening the connections among variables whose intensities, in relation to external circumstances, have significantly increased. This explains Keynes' preference for ordinary language over formalised language to deal with complex phenomena. Because it allows the economic reasoning to rest upon hypotheses that do not explicitly contradict the essence of its material, it ultimately paves the way to complex magnitudes to be used for practical purposes notwithstanding their lack of a common unit of measure. Therefore, even though the agent-based methodology is able to overcome, in a certain sense, the limitations of mathematical calculus by means of computer simulations which can be run thousands of times in order to produce large quantities of clean data and make science with complexity, it calls for another notion of complexity – 'computational complexity' – defined in terms of the minimum length of a computer program to describe the information of a system (see Rosser 2004-2005). This implies, in turn, a view of complexity focused on the property of the object observed and independent from that of the observer. To this extent, the 'Santa Fe perspective' has not properly understood the methodology of complexity as envisaged by Keynes, in which the open structure of ordinary language requires another level of complexity which concerns not only the object of the theory – the economy – but also relations between its object and subject – the economist. As Gotti (2009: 298) acknowledges, the interplay of theory and method renders "reader involvement" a necessary requisite of Keynes's work because it assigns to the readers "a far more demanding role as his collaborators in working out the final form and the exact meaning of a new economic theory".

In conclusion, despite the fact that agent-based models are certainly capable of accounting for the interdependencies of the real-world economy, which is what Keynes' complexity is aimed at, they implicitly rely upon a form of mathematical formalism that, even though it is not the same as that which Keynes condemned, makes it difficult to establish a direct *nexus* between the definition of complexity provided by the latter and that of the Santa Fe Institute. However, given the need to communicate the results of the analysis, science requires a certain degree of decomposability that, if the organic and the artist's synthetic perspective envisaged by Keynes' theory is adopted, poses a "problem of paralysis" (Chick 2003: 318). Therefore, if Keynes' complexity vision has not been able to survive within the evolving institutional framework of professional economics, this may be due to a presentation of his theory that was "heuristic" and did not deal with "how his ideas could be translated into a precise mathematical presentation" available at the time, given the complete absence of "expertise to formally model our economy as a complex system" (Colander 2001: 190). If our desire is to restore the validity of the 'Keynesian revolution' as a 'complexity revolution' which analyses the economy as a "complex adaptive system', we may have to abandon the dominant current economic theorising which "attempts to apply highly precise and mathematical methods to material which is itself much too vague to support such treatment" (Keynes 1973D: 379), and allow the "process-and-emergence perspective" to dethrone Keynes' interpretation provided by the reigning 'neoclassical synthesis'.

4. Concluding Remarks

In a social world made of agents who attempt to learn and to adapt their behaviour to emerging changes endogenously generated by the interactions of the political, institutional and economic actors, and where complete knowledge of its premises is certainly precluded, it is necessary to conceive it in terms of a complex adaptive system. This is closely related to the economy that Keynes portrayed in the *General Theory*, which shows how individuals do not rationally combine several hypotheses being completely aware that they will be the correct ones, and how an economy could get into trouble and end up in an undesirable equilibrium. Nonetheless, Keynes' failure to provide a satisfactory analytical treatment of his complex vision of the real world could be meaningfully considered within a highly complex dynamic framework, which might be reformulated by the 'Santa Fe perspective' in order to investigate those parts of Keynes's theory discharged by mainstream economics – the complex dynamics discussed in Section 3.

To conclude, insofar as we endorse the definition of 'dynamic complexity' discussed in the paper, we agree with the definition of Keynes as a "thinker of economic complexity" provided by Marchionatti (2010). However, because of the complexity involved not only in the properties of the world but also in the process of inquiry into the world, which involves the complex relation between the object and subject of economic theory, it might be interesting to investigate the extent to which Keynes' complexity can be compared to Delorme's "deep complexity" (2010), a transdisciplinary framework for addressing complex problems in which complexity denotes both the intricacy and the difficulty of dealing with it.

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