

TERENZIO COZZI AS A GROWTH THEORIST

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ABSTRACT

The present essay reviews the main contributions to the analysis of economic growth given by Terenzio Cozzi. Besides two textbooks, which provide a detailed overview of the various strands of the discipline during the 1960s and the 1970s, Cozzi proposes a few personal contributions that fit perfectly into the Cambridge post-Keynesian tradition of those years: they develop Luigi Pasinetti's model of structural economic dynamics along different directions and provides a multisectoral framework for analyzing the short run dynamics of the relation between growth and income distribution.

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INTRODUCTION

I first became acquainted with the work of Terenzio Cozzi as a student when reading *Teoria dello sviluppo economico* (1972a), which was one of the textbooks adopted by Siro Lombardini for the Economics II course he taught at Università Cattolica del Sacro Cuore in Milan during the 1983-1984 academic year. I remember Cozzi's textbook due to the relevance of the topics covered and the extremely clear manner in which they were presented. Most of my subsequent research draws from the subjects he

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addressed in the textbook. Upon completing my undergraduate degree, I received a one-year grant from the Fondazione Einaudi (in 1989) and then a post-doctoral research grant (in 1993).

I got to personally meet Terenzio Cozzi at Fondazione Einaudi, and continued to develop a professional and personal relationship with him through the Italian Economic Association (SIE), and the Italian Association for the History of Political Economy (STOREP) which he both presided (in fact, he was STOREP's first President). I have always found him to be an amiable, kind, helpful and cooperative colleague.

Let me turn to the subject of my paper: Terenzio Cozzi as a growth theorist. His contributions that specifically relate to this area are all concentrated in the early years of his career. The most relevant titles include:

1965, "Sulla convergenza al saggio di crescita naturale di un processo dinamico" [On the convergence to the natural growth rate of a dynamic process], in F. Forte and S. Lombardini (eds.), *Saggi di economia*, Milano: Giuffrè;

1966, *Movimenti in equilibrio nell'analisi macroeconomica* [Equilibrium dynamics in macro-economic analysis], Torino: Giappichelli;

1969, *Sviluppo e stabilità dell'economia* [Growth and stability of the economic system], Torino: Fondazione Luigi Einaudi;

1972, *Teoria dello sviluppo economico* [Theory of Economic Growth], Bologna: il Mulino;

1974, "Alcune considerazioni sull'andamento di breve periodo dei saggi di crescita e dei saggi di profitto in un modello plurisetoriale" [Some considerations on the short-term behaviour of growth rates and profit rates in a multisectoral model], *Rivista Internazionale di Scienze Sociali*, LXXXII (I-II).

1. A SYSTEMATIC EXPOSITION OF THE POST-KEYNESIAN THEORY OF GROWTH AND DISTRIBUTION

The volume *Movimenti in equilibrio nell'analisi macroeconomica*, published in 1966, is an erudite monograph of the issues that were being discussed at the time. The starting point are the problems emphasized by Harrod and Domar on how a growing economic system can maintain full employment and full capacity utilization in the long run. On the one hand, by extending the principle of effective demand to the long run and imposing the condition that investment raises production capacity in line with the evolution of effective demand, Domar identifies that the growth rate that guarantees this equilibrium growth is equal to the s/v ratio, called 'warranted rate of

growth', where s is the community's propensity to save and v is the capital/output ratio.¹ Harrod proves that this equilibrium growth path is *unstable*, i.e. if the system deviates from it for any reason, the imbalance would tend to grow larger instead of being reabsorbed.

More specifically, starting from a situation of dynamic equilibrium, if investments were to grow at a rate *higher* than s/v , the induced growth in demand would make the installed production capacity *lower* than what is required to satisfy demand. This would give rise to the paradoxical situation whereby entrepreneurs, despite having invested more than they should, find themselves with less production capacity than desired.² This imbalance is, of course, cumulative: to make up for the lack of production capacity, entrepreneurs would be pushed to invest more, thus accentuating the imbalance and giving rise to *inflationary tensions*. On the contrary, if, starting from a situation of equilibrium, investments were to grow at a growth rate *lower* than s/v , the induced growth in demand would make the installed productive capacity *higher* than what is required to satisfy demand; again, another paradox would arise since entrepreneurs, having invested less than they should have, would find themselves with a production capacity higher than desired.³ To offset the surplus of production capacity, entrepreneurs would then be pushed to invest even less, thus accentuating the imbalance and giving rise to a *cumulative depression*

¹ A simple formulation of the Harrod-Domar model can be given as follow (for our purposes it is better to argue in discrete terms). Investments exert a twofold effect on the system: *i*) they increase effective demand according to the multiplier relation,

$$Y_t = (1/s)I_t, \tag{M}$$

and increase productive capacity by

$$P_{t+1} - P_t = (1/v)I_t, \tag{PP}$$

where Y_t , I_t , and P_t denote output, investments and productive capacity referred at period t , s is the marginal propensity to save and v is the capital/output ratio. To identify the equilibrium path relation (M) must be set in dynamic terms:

$$Y_{t+1} - Y_t = (1/s)(I_{t+1} - I_t), \tag{MM}$$

Let us suppose that at $t = 0$ demand and productive capacity coincide, that is, $Y_0 = P_0$. The system evolves along an equilibrium path (that is, demand increases at the same pace of productive capacity) if $Y_{t+1} - Y_t = P_{t+1} - P_t$ which, thanks to (PP) and (MM), entails

$$\frac{I_{t+1} - I_t}{I_t} = \frac{s}{v}. \tag{HD}$$

² In terms of the formulation given in footnote 1, the equilibrium level of investments is $I_{t+1} = I_t \left(1 + \frac{s}{v}\right)$. If the system is in dynamic equilibrium at t , then $Y_t = P_t$; suppose that at $t = 1$ investments are fixed at $I_{t+1} = I_t \left(1 + \frac{s}{v} + \varepsilon\right) > I_{t+1}$, then we have from (MM) $I_{t+1} = P_t + \frac{1}{v} I_t + \frac{\varepsilon}{s} I_t$ and from (PP) $P'_{t+1} = P_t + \frac{1}{v} I_t$. By comparing these formulas, we deduce $Y_{t+1} > P'_{t+1}$: thus, having invested more than the equilibrium level, production capacity is insufficient to meet demand.

³ A similar, yet opposite reasoning to the one described in footnote 2 applies here.

phase.⁴ To underline the unstable nature of Harrod's equilibrium path, it has been referred to as 'knife's edge growth equilibrium path'.

Things become even more complex if, following Harrod, we also consider the growth rate of labour-power, g_n ; in this way, the growth that guarantees the full employment of productive capacity *and* labour-power requires that $s/v = g_n$. Since, however, all three quantities involved in this relationship are given, they can satisfy it only by a fluke. Two situations may thus arise.

- i) If $s/v > g_n$, the labour force will grow at a rate lower than the warranted rate (that which would guarantee effective demand growing at the rate of growth of productive capacity); if there is a margin of unemployment, the system could initially grow at the warranted rate, however, once unemployment is absorbed, the system will have to grow at the natural rate, which will then act as a bottleneck. But, due to the knife's edge instability of the equilibrium growth path, if the system grows below the warranted rate, the installed productive capacity turns out to be higher than the level required to satisfy demand. Entrepreneurs will consequently decide to invest less than they would otherwise have done; aggregate demand will grow even less, and the economic system will find itself in a phase of cumulative depression with the consequent phenomena of unemployment. In this case of $s > vg_n$ the system's propensity to save is greater than the share of income needed to finance the investments necessary to make the system grow in the long run at the rate that would ensure full employment. This results in unemployment due to a lack of effective demand, i.e. of a Keynesian nature.
- ii) If $s/v < g_n$ the labour force will grow at a higher rate than the guaranteed growth rate. Thus two sub-cases could arise: ii-a) the system grows at the natural growth rate, i.e. at a rate higher than the guaranteed growth rate, thus giving rise to inflationary tensions, due to the knife's edge instability outlined above; ii-b) the system grows at the guaranteed growth rate: there would be full employment of the productive capacity but 'structural' or 'classical' unemployment of the labour force, caused this time by a shortage of capital needed in order for the available labour force to work.

⁴ It must be said that Harrod's description of this process is much less schematic than described here: he recognizes that small perturbations from the equilibrium path do not necessarily give rise to the cumulative divergence phenomena outlined above; moreover, he admits that some possible changes in parameters s and v can (partially) offset these divergent forces.

At least two ways to overcome this *impasse* have been proposed by economists. According to neoclassical economists, the disequilibrium would set in motion some specific market mechanisms that would modify parameter v , which measures the capital intensity of the techniques adopted, in the appropriate direction to achieve the Harrod Domar equality. According to the Neo-Keynesians, an appropriate redistribution of income could be obtained (sometimes spontaneously, through price flexibility, sometimes through appropriate redistributive policies) that modifies parameter s in the direction of achieving equality. An extensive and detailed review of the neoclassical and neo-Keynesian literature on the subject is presented in Cozzi (1966). Moreover, the book provides a methodological systematization – truly valuable for those times – of a whole series of concepts relevant to the discussions, such as the notion of equilibrium, the difference between static and comparative dynamic analyses, the notion of equilibrium stability, the convergence to a particular equilibrium path, and the role of expectations. This systematization of concepts and clarification of the context and purposes pursued in the various contributions reviewed reflects the imprint of his many mentors, such as Carlo Felice Manara, Richard Goodwin, Joan Robinson and, above all, Siro Lombardini.

In sum, the material presented in the volume is certainly broad and deep, comparable in completeness and accuracy to the famous essay by Hahn and Matthews (1964), at least when considering the part on macro-economics. Nonetheless, it is not an original contribution to the analysis of growth, that will occur in the following essay.

2. GROWTH AND STABILITY OF THE ECONOMY

In 1969, Cozzi finalized his book *Sviluppo e stabilità dell'economia*, which was published by the Luigi Einaudi Foundation. It is the Italian version of the research he undertook at Cambridge for his PhD dissertation 'A Disaggregated Model of Economic Growth and its Stability'. The book constitutes an original in-depth study and development of Luigi Pasinetti's model of structural economic dynamics.⁵

It is worthwhile briefly going over Pasinetti's analytical scheme in order to frame the context in which Cozzi operated. Pasinetti's framework is

⁵ In turn, the structural economic dynamics model constitutes the content of Pasinetti's PhD thesis (PASINETTI 1962), defended at Cambridge University in March 1963. Initially only the first five chapters of his thesis were published in a typescript by King's College (PASINETTI 1963) and later in a volume by the Pontifical Academy of Sciences (PASINETTI 1965). The entire contribution was published only twenty years later: PASINETTI 1981.

certainly a groundbreaking work. Certainly, it represents a break with respect to mainstream analysis: he considers a production model in which all phenomena of scarcity of factors of production are left aside and in which the production units observed are industries or sectors and not individual firms. His theory also breaks with respect to the heterodox approach put forth during those years. However, it is not a Marxist model. It starts from input-output analysis, but almost immediately gets rid of the 'straitjacket' of industrial relations analysis (Pasinetti 1993) and re-proposes an analysis in vertical integrated terms, following the methodology set forth in Pasinetti (1973), which is based on the notion of sub-systems as proposed by Sraffa (1960, Appendix A). Furthermore, Pasinetti keeps behavioral relations to a minimum: he leaves out, at least in the first instance, the study of the forces that lead the economy towards equilibrium.⁶ Furthermore, he distinguishes a set of *sectoral* 'equilibrium' conditions (demand = supply and price = production expenses plus a possible percentage mark-up on the value of the means of production in each of the sectors) from the *macro-economic* equilibrium condition (full employment of the labour force and full expenditure of distributed incomes – that is, wages and profits – in consumption and investment). This distinction allows him to give substance to the Keynesian notion of under-employment equilibrium in a multisectoral model. More specifically, equilibrium can be reached at the sectoral level while there is disequilibrium at the macro-economic level. Moreover, he considers these as *necessary conditions*, that is, conditions that *have to be* satisfied in order for the system to be in its 'natural' equilibrium, that is, an ideal situation where the system's potential regarding satisfaction of final wants, growth and employment are realized at their best. No automatic convergence to this configuration is thought to operate in real economies. On the contrary, this 'natural system' is regarded by Pasinetti as a 'norm' to be considered as a reference point, or a goal to be pursued or, at least, approached by the institutions. While Pasinetti's framework has often been appreciated for the *description* it gives of the phenomenon of structural change, its normative nature, the methodology proposed, and several implications of his analysis have often been disregarded.

⁶ It should be clarified that the very term 'equilibrium' in Pasinetti's analysis (as well as in Sraffa's and in the modern classical approach) is not entirely appropriate: it surely does not refer to 'market equilibrium', that is, to a situation where a set of prices prevail at which the quantities of goods and the productive factors demanded and supplied coincide. The equilibrium solutions in PASINETTI'S (1981) model include: *i*) equalities between supply and demand for the various commodities; *ii*) equalities between prices and expenses of production plus a rate of profit (not necessarily uniform among sectors); *iii*) a macro-economic condition, that is, a condition that the technical and consumption coefficients have to satisfy in order to be compatible with full employment of labour.

Terenzio Cozzi grasps Pasinetti's analysis quite well. He deepens various aspects of the model and provides some interpretations of his own. His methodology of analysis is partially different from Pasinetti's, who considers continuous changes in all the parameters of the model (labour force, individual final demand of each good, labour coefficients). Cozzi instead analyses the various phenomena by considering *una tantum* discrete changes of one parameter at a time and studies the various steps that must occur at the sectoral and at the macro-economic levels for the system to adjust to such changes. The main part of the results is drawn by using a model with only two final commodities.

In Chapter I, Cozzi studies the consequences of each of the following phenomena:

- i) the occurrence of technical progress in one of the two sectors;
- ii) proportional and non-proportional consumption growth; the phenomenon of consumption saturation;
- iii) the introduction of a new good;
- iv) the reduction of labour time;
- v) technical progress, which is here *endogenized* (it is put in direct connection with the growth of final demand);
- vi) technical progress incorporated in new capital goods.

Of particular interest, for our purposes, are the last two points, which deserve a little more in-depth consideration. Regarding the endogenization of progress (item v) Cozzi, following Verdoorn (1949), assumes that the labour coefficients of the generic commodity i and of the productive capacity of commodity i decrease at rates that depend positively on the rate of increase of production of commodity i and on the rate of increase of production of its productive capacity,

$$-\frac{\dot{a}_{ni}(t)}{a_{ni}(t)} = n_i + d_i \frac{\dot{x}_i(t)}{x_i(t)} \text{ and } -\frac{\dot{\alpha}_{ni}(t)}{\alpha_{ni}(t)} = v_i + \delta_i \frac{\dot{\xi}_i(t)}{\xi_i(t)}, \quad (1)$$

where, the roman letters refer to final goods and the Greek letters to capital goods $a_{ni}(t)$ and $\alpha_{ni}(t)$ are the quantity of labour necessary to produce 1 unit of final good i and 1 unit of its productive capacity, $x_i(t)$ and $\xi_i(t)$ are the total quantity produced of final good i and of its productive capacity (1 unit of productive capacity of final good i is the complex of capital goods necessary to produce 1 unit of final good i); finally, n_i , d_i , v_i and δ_i are positive coefficients. According to formulas (1) labour input coefficients are positively affected by the increase of produced quantities: the more the output increases, the more technical coefficients decrease, that is, the

more the sectoral labour productivities increase. Technical progress is thus no more exogenous, like in Pasinetti's original formulation, but it remains induced by the increase of the specific final demand. In item *vi*), Cozzi considers the case of technical progress embodied in new capital goods. He denotes by $a_{ni}^*(t)$ the labour input coefficients when labour is equipped with the new productive techniques. Thus, $a_{ni}^*(t) \leq a_{ni}(t)$, and the actual labour coefficient is a suitable weighted average of these coefficients. Throughout the first chapter, the author mainly considers the ideal path of the quantities produced for the system to grow in full-employment equilibrium.

The second chapter is devoted to prices, income and consumption. Following Pasinetti, Cozzi assumes that each final good is produced by means of labour and a (composite) capital good that in turn is produced by means of labour alone. For the sake of simplicity, production is here regarded, in Sraffa's words, as a 'one-way avenue', that leads from labour to final goods; in other terms, this simplifying assumption excludes circularity in the production process.⁷ Yet, once a *numeraire* is chosen, one distributive variable must be fixed from outside the price system. It is instructive to note the path followed by Cozzi, which is not connected to Pasinetti's notion of 'natural rates of profit'. In fact, although developed in his PhD thesis (Pasinetti 1962), this notion had not been published yet when Cozzi wrote his book.⁸ Cozzi, in order to determine the rate of profit, closely followed Kaldor's and Pasinetti's aggregate theory of income distribution, according to the particular interpretation in normative terms proposed by Pasinetti. Cozzi assumes that workers consume their wages entirely while capitalists save their profit entirely (a relaxation of these extreme assumptions does not alter the main conclusions); in this way an increase in the rate of profit entails an increase in savings. He then identifies the level of the rate of profit which ensures that the global investments needed for the economy to grow in equilibrium and in full employment are equal to global savings. Given the strict assumptions on the propensities to save, Cozzi shows that the rate of profit is equal to the rate of growth of the system. This result replicates the equality obtained in von Neuman's growth model. Yet, since the rate of growth of the system can vary across time, the rate of profit can vary in the long run as well, according to the shocks occurring in the system. Clearly, we are still at a *normative* level of analysis. On this basis, Cozzi investigates

⁷ Pasinetti also considers the case where capital goods are involved in the production of capital goods (see PASINETTI 1981, chapter II, § 7 and PASINETTI 1988). However, the main conclusions of his analysis do not change significantly with this generalization.

⁸ The natural rates of profit are introduced by Pasinetti in Chapter VII of his thesis. This chapter was thus not available in 1969, when Cozzi wrote his book.

the effects of some of these shocks separately. Let us consider one of these analyses. For example, let's consider the scenario where technical progress consists of an *una tantum* reduction in the labour coefficient of one industry. For the sake of simplicity, let us start by considering the case of a system of proportional dynamics. The maintenance of full employment requires a transitory increase of the rate of profit to relocate the workers expelled from the sector that has experienced technical progress towards the sectors producing the capital goods. This would be in fact, according to Cozzi, the way to relocate the expelled workers since the production of final goods depends on final demand, and there would be no way to increase the latter after technical progress has occurred, because employment can increase only if the production of capital goods increase. The absorption of this additional output of capital goods requires an increase of savings, which can be obtained thanks to the increase of the rate of profit. Subsequently, the sectors producing final goods must increase their output at higher rates than those producing capital goods in order to absorb the additional capital goods produced. This requires a subsequent reduction of the rate of profit, which can return to the initial level. Quite similar results hold if the system enjoys structural dynamics. In addition, the effects of technical progress occurring in one sector are analyzed on prices, on consumption and on investments. In terms of labour, the price of the good produced in the industry in which technical progress occurs decreases with respect to price of the other good. Consumption can increase in real terms thanks to technical progress; investments remain constant due to the assumptions that capital goods are produced by means of labour alone and full employment is maintained in our logical experiment.

Lastly, in the final part of Chapter II, Cozzi proposes an endogenization of consumption coefficients: an aspect apparently inescapable to mainstream economics, for which it is inconceivable that consumption coefficients, that is, final demand, is independent of relative prices. Cozzi thus introduces in Pasinetti's model the following per capita final demand for each good i by the 'representative consumer':

$$a_{in} = \varphi_i \left(y, \delta, \frac{p_2}{p_1} \right),$$

where the average y is per-capital income and δ is the global wage share. This entails that:

$$\frac{\dot{a}_{1n}}{\dot{a}_{2n}} = F \left[y, \delta, \frac{p_2}{p_1}, \dot{y}, \dot{\delta}, \frac{d}{dt} \left(\frac{p_2}{p_1} \right) \right].$$

According to Cozzi, the kinds of changes which are relevant in actual growth processes often reflect inertia phenomena or occur when some variables reach a certain threshold value level. Then behaviour is best captured by discontinuous changes of demanded quantities as long as income, prices, or the wage share change. Subsequently, the introduction of new goods (both capital goods as addressed above, but also final goods, or goods that can be used either as capital or as final goods) is addressed: the case of new capital goods is easier to consider since their introduction brings about a reduction in production costs, while the introduction of new goods that are (partially or totally) consumption goods reflect changes in consumer preferences. Cozzi then returns to the phenomenon of technical progress as induced by the dynamics of sectoral final demand: a good characterized by an active dynamic of final demand induces a higher reduction in labour coefficients (a higher increase in labour productivity) and thus of prices. The latter accelerates the dynamics of final demand, thus approaching the saturation level and dampening the whole process of induced innovation.

In Chapter III Cozzi investigates, from multiple angles, the stability properties of the model. He first analyzes *sectoral imbalances*. Adhering to a Keynesian view, Cozzi assumes that as long as the system is not in full employment, sectoral imbalances regarding quantities are reabsorbed by the system by adjustments based on quantity- rather than price-flexibility. According to a 'stock-adjustment principle', the productive capacity of commodity, x_i , varies according to the following differential equation:

$$\dot{x}_i = \dot{D}_i^* + \sigma_i(D_i - x_i), \quad \sigma_i > 0,$$

where D_i^* is expected demand and D_i is the actual demand of commodity i : the productive capacity of commodity i is driven by the expected demand for that commodity and reacts to the imbalances between actual demand and productive capacity. Sectoral imbalances are studied when the macro-economic condition is 'less-than satisfied', that is, the system exhibits a degree of unemployment (Section 4). In particular, if the demand for a commodity exceeds its productive capacity, this latter tends to adapt to the demand for that commodity, and the process could bring the labour force to full employment if global demand is not lower than the level corresponding to full employment. If, on the contrary, we start from a situation of full employment, the results change depending on whether the nominal wage is constant (Section 5) or can vary (Section 6). Since there is no unemployment, the excess of demand of one or more commodities will result in price increases. In this case, if nominal wages are constant (for example because workers are not sufficiently strong to prevent a real wage reduction) the excess of demand will be reabsorbed. If, on the contrary, nominal wages

are variable, that is workers are able to keep real wages unchanged (which is quite likely for a system at full employment) the outcome is not univocal; the results lie between two poles depending on power relations between capitalists and workers: convergence to full employment equilibrium or the emergence of an inflationary spiral. Finally, in Section 7, Cozzi considers the case where global demand is not sufficient to maintain full employment. His analysis confirms Pasinetti's findings: there are no automatic mechanisms that guarantee the convergence of the economic system to full employment equilibrium; specific remedies must be identified, each time different, depending on the circumstances of the system.

The merit of Cozzi's inquiry was that of inserting various phenomena typical of modern economies within the framework of Pasinetti's model. He not only pioneered such a difficult task but also achieved impressive results. However, there are a couple of open points that need further reflection. First, Cozzi, following Pasinetti, carries out his analysis in vertically integrated terms (Cozzi 1969: 24). For the sake of simplicity, he considers two consumption goods. Each consumption good requires the employment of labour and a specific (composite) capital good which, in turn, is produced by only employing labour. This implies that all the comparative static analyses as well as the adjustment processes discussed above take place within or between vertically integrated sectors. Nevertheless, this point is disputable and requires further discussion. Secondly, in Chapter III Cozzi studies the dynamics of the economy *outside* of the equilibrium positions described by Pasinetti's model. However, Pasinetti's focus is on the dynamics *of* these equilibrium positions.⁹ Two dynamics operate thus jointly. An open question remains whether it is more useful to study these dynamics separately or jointly (a quite isolated attempt to study them jointly was proposed by Duménil and Lévy 1995).

3. SHORT PERIOD TRENDS IN GROWTH AND PROFITS IN A MULTISECTORAL MODEL

Cozzi published a further paper on growth theory in 1974 in the *Rivista internazionale di scienze sociali*. The topic of this paper is quite different from Pasinetti's long-run analysis. It can be seen as an attempt to provide a multisectoral framework for analyzing the short run dynamics of the relation between growth and income distribution. The starting point is a relationship that Cozzi proved in a previous paper (Cozzi 1972b) according

⁹ Concerning the meaning to be attributed to the term 'equilibrium' in Pasinetti's analysis, see what stated in our previous footnote 3.

to which in (macro-economic) equilibrium the weighted average of sectoral rates of growth equals the weighted average of sectoral rates of profit multiplied by the propensity to save of capitalists. In fact, by re-writing the macroeconomic equilibrium conditions, $I = S$, in disaggregated terms we have $\sum_{i=1}^N I_i = \sum_{i=1}^N S_i$, which can be re-expressed as $\sum_{i=1}^N \frac{I_i K_i}{K} = \sum_{i=1}^N \frac{s_c P_i K_i}{K}$ or as

$$\sum_{i=1}^N g_i h_i = s_c \sum_{i=1}^N r_i h_i, \quad (2)$$

where $g_i = I_i/K_i = \dot{x}_i/x_i$ is the growth rate of capital and of output in sector i (we are supposing a constant capital/output ratio α_i for each sector, that is $K_i/x_i = \alpha_i$), $r_i = P_i/K_i$ is the rate of profit of sector i , $h_i = K_i/K = (\alpha_i x_i)/(\sum_{i=1}^N \alpha_i x_i)$ is the 'weight' of sector i in terms of its capital over total capital and s_c is the propensity to save of capitalists. If Cozzi's equality (2) denotes a situation where macro-economic equilibrium prevails, the difference between the two members of (2),

$$E = \sum_{i=1}^N h_i (g_i - s_c r_i), \quad (3)$$

expresses the *macro-economic* imbalance between investment and savings (per unit of capital) as a function of the *sectoral* imbalances between investment and savings (per unit of sectoral capital),

$$z_i = g_i - s_c r_i, \quad i = 1, 2, \dots, N. \quad (4)$$

Cozzi observes that the g 's may vary depending on the level of idle capacity, the state of entrepreneurial expectations and, above all, the overall excess demand. On the other hand, the r 's can also vary depending on the degree of competition in the various sectors, the level of idle capacity, the bargaining strength of workers, and consequently, the level of unemployment and, above all, on overall excess demand. Since Cozzi considers the effect of the overall excess demand on both the g 's and the r 's to be preponderant over the others, he supposes

$$\dot{g}_i = \eta_i E \quad i = 1, 2, \dots, N, \quad (5)$$

$$\dot{r}_i = \gamma_i E \quad i = 1, 2, \dots, N, \quad (6)$$

where η_i and γ_i are two sets of constant parameters. The other possible effects will be considered at a second stage of the analysis, in non-formal terms, by qualifying the direction or the intensity of some of the analytical results derived. Let us substitute (5) and (6) into the expressions of the variables z_i defined in (4); we have $\dot{z}_i = \dot{g}_i - s_c \dot{r}_i = (\eta_i - s_c \gamma_i) E$ that is,

$$\dot{z}_i = (\eta_i - s_c \gamma_i) \sum_{j=1}^N h_j z_j, \quad (7)$$

or, in compact form,

$$\dot{\mathbf{z}} = \hat{\boldsymbol{\varepsilon}}\mathbf{H}\mathbf{z}, \tag{7'}$$

where $\dot{\mathbf{z}}$ is the column vector of time derivatives of the z_i , $\hat{\boldsymbol{\varepsilon}}$ is a diagonal matrix whose generic element is $\varepsilon_i = \eta_i - s_c\gamma_i$, $i = 1, 2, \dots, N$, and

$$\mathbf{H} = \begin{bmatrix} \mathbf{h}^T \\ \mathbf{h}^T \\ \vdots \\ \mathbf{h}^T \end{bmatrix},$$

is a square matrix with all rows equal to $\mathbf{h}^T = [h_1, h_2, \dots, h_N]$. (7) is a linear differential system whose state variables are the z_i s. The system has certain analytical properties that deserve a few comments. The steady state conditions, $\dot{\mathbf{z}} = \mathbf{0}$, reduces to

$$\mathbf{h}^T \mathbf{z} = \mathbf{0}, \text{ or } \sum_{j=1}^N h_j z_j = 0 \tag{8}$$

that is, *one equation in N unknowns*: z_1, z_2, \dots, z_N . This means that there is an *entire set* of steady states constituted by a linear hyperplane with dimension $N - 1$.¹⁰ \mathbf{H} is a rank-one matrix, as is matrix $\hat{\boldsymbol{\varepsilon}}\mathbf{H}$ (the pre-multiplication of \mathbf{H} by a diagonal matrix yields a matrix where each row is a multiple of the first one). Hence, the eigenvalues of the differential systems are all zeros except for one, which is equal to

$$\lambda^* = \sum_{i=1}^N (\eta_i - s_c\gamma_i)h_i.$$

The dynamic properties of the state variables z_i depend on the sign of λ^* : if $\lambda^* > 0$ the z_i s diverge exponentially from the steady state manifold defined by condition (8); alternatively, if $\lambda^* < 0$ the z_i s converge exponentially to the

¹⁰ To outline this result, Cozzi uses the term ‘neutral equilibrium, meaning that the economic system shifted from an equilibrium position neither returns to its starting position nor tends to move increasingly away from it. Instead, the system tends towards a new equilibrium position that is more or less removed from the starting position’ (Cozzi 1974: 79, own translation). To provide an example, observe that with $N = 3$ condition (8) turns into $h_1 z_1 + h_2 z_2 + h_3 z_3 = 0$; in this case, two values of z_i can be fixed at will, for example, $z_1 = g_1 - s_c r_1$ and $z_2 = g_2 - s_c r_2$, and the remaining one must satisfy

$$g_3 - s_c r_3 = (h_1/h_3)(g_1 - s_c r_1) + (h_2/h_3)(g_2 - s_c r_2).$$

This overall link among the rates of variations of the sectoral excesses of demand repropose the idea of a sort of ‘frontier’ for the sectoral rates of growth mentioned by Cozzi on p. 84.

steady state manifold. Cozzi devotes a portion of the paper to discussing the sign of λ^* . And it is precisely here where the considerations of the various extra-analytical effects mentioned above come into play. In short, he argues that most of the η_i parameters are positive; at the same time there are a number of economic considerations that lead one to also expect that the γ_i parameters are positive. This result is thus quite indeterminate, but as recalled in the title of the paper, we are carrying out a *short period* analysis; hence the interest is not in the asymptotic properties of the state variables. If the system starts from a position quite close to the steady state, a λ^* eigenvalue not too far from 0 (even if positive) does not excessively displace the state variables from the steady state manifold. Moreover, outside the formal model, Cozzi details several reasons why the levels and the signs of parameters η_i and γ_i modify according to the sign of the overall excess of demand E and, more in general, during the expansive or the contractive phases. On the whole, it can be said that the analytical model presented in the paper can be regarded as a starting point to develop a broader analysis. The analytical model itself, shall we say, is under-exploited. With suitable additional restrictions it could provide more definitive and relevant results.

CONCLUDING REMARKS

In summary, we can say that Terenzio Cozzi's contributions to the theory of economic growth are stimulating and challenging. The two monographs considered in this paper provide a very detailed and updated evolution (with respect to the times) of the theory both for undergraduate students (Cozzi 1972a) and postgraduate students (Cozzi 1966). As was customary for Italian economists at the time, well-defined theoretical stances were adopted but ample room was allowed for the theoretical pluralism that existed in the discipline.

Cozzi's specific research path was clearly positioned within the post-Keynesian theoretical strand, to which he provided very original and relevant contributions. In Cozzi (1969) he worked out several personal extensions and insights into Pasinetti's model of structural change when only the basic structure of the model had been published at the time. In this way Cozzi developed the structural change model along directions that did not always coincide with those worked out by Pasinetti (see, for example, the determination of the rate of profit proposed by Cozzi) and covered some aspects that were often regarded as shortcomings of Pasinetti's model. Some questions still remain open today, especially with reference to the possibility to study the adjustment processes, given the vertically (hyper-)integrated nature of the sectors considered in Pasinetti's model.

Cozzi, however, never returned to these topics. The material presented in Cozzi (1974) provides a different perspective: it pushes post-Keynesian theory of growth along new lines. The analytical model, as recalled above, is deliberately generic, in order to act as a sort of ‘container’ to be specified in more detailed analyses from time to time: it provides many arguments, specifications and generalizations, which should or could be made to interact directly with the dynamics described by the analytics of the model. A task that still has to be done!

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